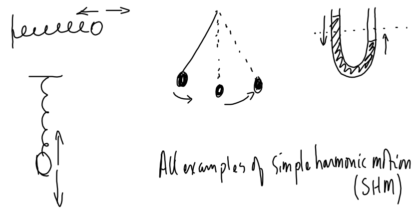
Oscillatory Motion

Glider on air track

oscillatory but not simple harmonic motion
(no net force, except at ends)

Bob on a spring: Pendulum: Tube of water:



All examples of simple harmonic motion (SHM)

There is an unbalanced or net force acting on the object when the object is displaced from its equilibrium position.

Recall Hooke's Law:

$$F = -kx$$

The unbalanced force in SHM is proportional to the displacement from the equilibrium position.

Period (T) => $T = \frac{\text{time}}{\text{cycles}}$ (s)

frequency (f) => $f = \frac{\text{cycles}}{\text{time}}$ (Hz)

Period + frequency are reciprocals

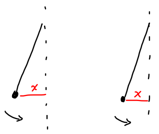
$T = \frac{1}{f}$ and $f = \frac{1}{T}$

$kHz = 10^3 Hz$

$MHz = 10^6 Hz$

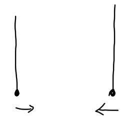
$GHz = 10^9 Hz$

Phase:

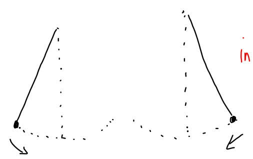


in phase.

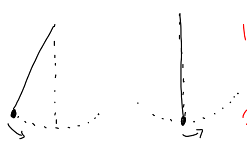
Phase Difference (out of phase)



out of phase



in opposite phase



The second pendulum leads the first and is a quarter of a period ahead

phase difference is $\frac{T}{4}$

Example

The atoms in an O_2 molecule oscillate with a frequency of 4.0×10^{14} Hz. What is the period of oscillation?

2.5×10^{-15} s

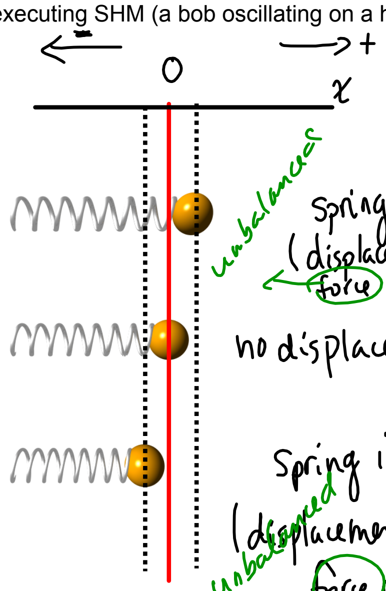
$$T = \frac{1}{f}$$

$$T = \frac{1}{4.0 \times 10^{14} s^{-1}}$$

$$T = 2.5 \times 10^{-15} s$$

So what is simple harmonic motion and what is the defining equation?

Consider the force acting on a body executing SHM (a bob oscillating on a horizontal spring, for example):



Bob is to the right of equilibrium — spring is stretched (displacement is to the right (+), force is to the left (−)) — unbalanced

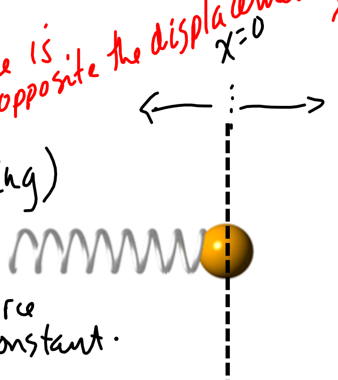
Bob is at equilibrium — no displacement / force is zero — balanced.

Bob is left of equilibrium — Spring is compressed (displacement is to the left (−), force is to the right (+)) — unbalanced

Recall Hooke's Law -> force is proportional to the displacement from equilibrium.

Acceleration of a body executing SHM:

force is opposite the displacement from equilibrium.



$F_{net} = -kx$ (restoring force)

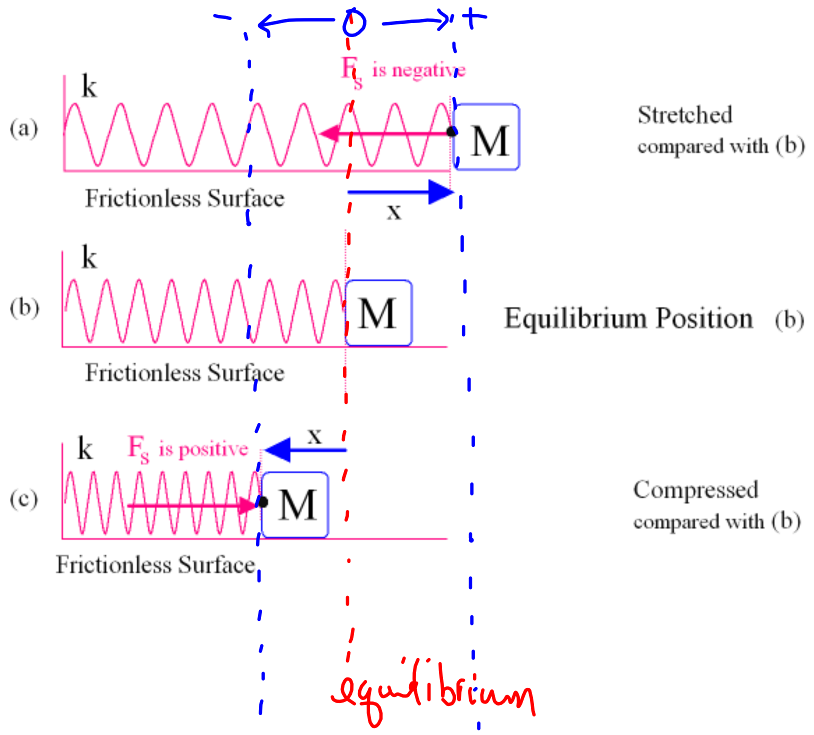
Where k is the spring or force constant.

Recall Newton's 2nd Law: $F_{net} = ma$ where m is the mass of the bob and a is the acceleration.

$ma = -kx$

$$a = \frac{-k}{m}x$$

← acceleration is directly proportional to the displacement and is in the opposite direction to the displacement. (i.e. towards the equilibrium)

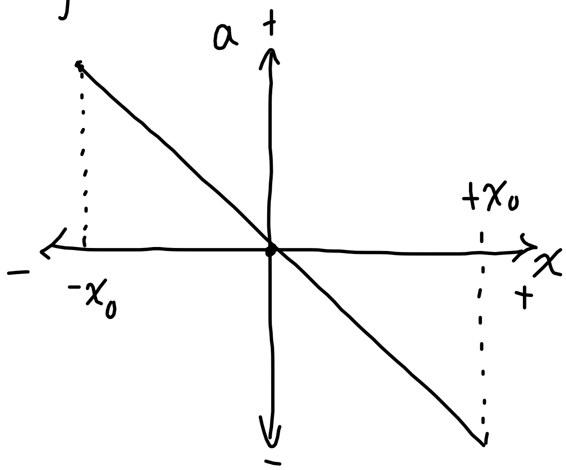


x is positive (to right)
F is negative (to left)
a is negative (to left)

$x = 0$, $F = 0$, $a = 0$

x is negative (to left)
F is positive (to right)
a is positive (to right)

A graph of acceleration vs extension



The graph is linear since the acceleration is directly proportional to x (displacement)

Definition of Simple Harmonic Motion

SHM is oscillatory motion in which the acceleration is:

- proportional to the displacement and
- is directed toward the equilibrium.

Since the acceleration is directly proportional and in the opposite direction we can write a proportionality statement:

$$a \propto -x$$

$$\boxed{a = -\omega^2 x}$$

← Defining Equation for SHM

Where x is the displacement
a is the acceleration
ω^2 is the proportionality constant

ω is called the angular frequency of the oscillation
units: s^{-1} or rad s^{-1}

more about the significance of ω^2 next week

In this case (i.e. bob attached to the horizontal spring):

$$a = -\frac{k}{m} x$$

$$a = -\omega^2 x$$

so $$\omega^2 = \frac{k}{m}$$

Note:

- We cannot solve kinematics problems involving SHM using our "suvat" equations because the acceleration is NOT constant during SHM. The acceleration is constantly changing!
- the significance of the kinematics graphs (x-t, v-t, a-t) are still valid

$$\begin{tikzcd}
x\text{-}t \arrow[r, bend left, "\text{slope}"] & v\text{-}t \arrow[r, bend left, "\text{slope}"] \arrow[l, bend left, "\text{area (displacement)}"] & a\text{-}t \arrow[l, bend left, "\text{area } (\Delta v)"]
\end{tikzcd}$$

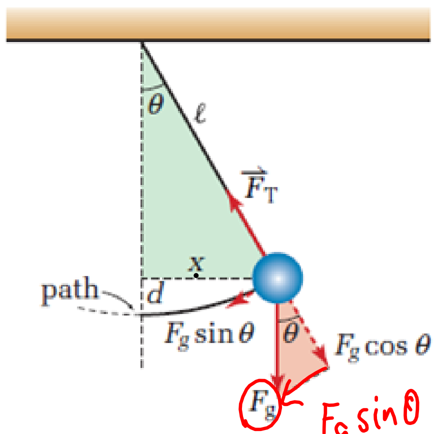
To show that an oscillatory motion IS simple harmonic motion, then we must show that:

$$a \propto -x$$

Consider the pendulum:
It has oscillatory motion, but is it SHM??

restoring force

$$F_{net} = mg\sin\theta$$



$= mg\cos\theta$

$F_g \sin\theta = mg\sin\theta$

mg

$$F_{net} = mg\frac{x}{\ell}$$

$$\cancel{m}a = \frac{\cancel{m}g}{\ell}x$$

$$a = -\frac{g}{\ell}x$$

insert −ve since acc is opp x

recall: $a = -\omega^2 x$ ← since the acceleration of the pendulum fits the defining equation for SHM, then its oscillatory motion is, infact, SHM.

$$\therefore \omega^2 = \frac{g}{\ell}$$

$$v = \omega x \qquad \omega = \frac{v}{x}$$